ESD-TDR-63-238 W-5490

A GENERAL METHOD TO COMPUTE THE CHARACTERISTICS OF MICROSTRIP TRANSMISSION LINES

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-238

May 1963

J. W. S. Liu

Prepared for

DIRECTORATE OF SYSTEMS DESIGN
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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THE MITRE CORPORATION
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ABSTRACT

A general method to compute the parameters of microstrip transmission lines is described. Here, the values of the potential function are estimated by solving the Laplace equation using the Monte Carlo method. The electric field strength is then computed by differentiating the potential function. From the electric field distribution, the charge density on the surface of center conductor, the associated capacitance per unit length of the line, the characteristic impedance, and the conductor loss of the line is computed. A Fortran program has been prepared and the flow chart of the program is included and described in this paper. The program was used to estimate the characteristic impedance of lines of three different geometrics. Results are within ten percent of the experimentally measured values.

INTRODUCTION

This paper is concerned with a general method to compute the characteristic impedance and conductor loss of microstrip transmission lines consisting of either a conductor strip above an infinite ground plane or a strip between two ground planes. It has been proven, both experimentally and theoretically, that for all practical purpose we may assume that the conductors are perfect and are imbedded in homogeneous and lossless dielectric material. Therefore, TEM mode may propagate in these types of lines. Thus the analysis of the parameters of these lines is essentially reduced to the investigation of the electric field distribution in the transverse cross-sectional plane of the line. From the field distribution, the charge density on the surface of the conductor, the capacitance per unit length of the line, and the characteristic impedance of the line can be determined. This problem is, therefore one of static electromagnetism.

However, even after this simplification is made, to derive an analytical expression for the parameters of such microstrip lines is still rather difficult. By making approximations in one way or another, many people have worked out solutions which give the parameters of lines with specific geometries. For the lines where the width of the conductor strip (hereon denoted as B) is large comparing with the spacing between the strip and the ground plane (hereon denoted as H), e.g., $B \geq 3.0 H$, the edge effect is negligible at the center of the strip. This case has been solved by using the conformal mapping method. However, in the cases where B is not large compared with H, the solution thus obtained are certainly not accurate. This problem has also been solved using conformal mapping method for any ratio of B and H; however in these instances, the

thickness of the conductor D is assumed to be negligibly small compared with other dimensions.

The analytical solution for the most general case is just too complicated, especially since we want this solution to be valid for both cases of open strip lines and sandwich lines. In view of the other approximation methods, it seems to be most suitable to solve our problem by solving the two dimensional Laplace's equation numerically since a computer program may be prepared for the most general case so as to include all possible geometries of the line.

Theoretically, in our problem, the boundaries includes points at infinity. Because only at these points do the fields due to the charges on the conductor strip vanish. However, at a distance large enough away from the strip, the electric field is practically zero. Therefore, we can approximate the boundary conditions as that shown in Fig. 1, where α and β are large numbers so chosen that at $y = \alpha$ and $x = \pm \frac{\beta B}{2H}$, the field strength is practically zero. Also, since all parameters we want to compute except the power loss per unit length of the line, are functions of the ratios B/H and D/H, no generality is lost by letting H = 1. Then, for any given H, the power loss per unit length can be easily obtained by multiplying the power loss obtained from this geometry by the proper multiplication factor.

The surface charge density on the center conductor and the associated capacitance can be determined from the field distribution just outside of the strip. From the value of the capacitance and the charge density, the characteristic impedance and power flow may be determined. Therefore, it is only necessary to find the electric potential at points close to the center conductor. For this reason, the

Monte Carlo method was chosen to solve the Laplace equation, because this method enables us to obtain the solution of the Laplace's equatic at these points without having to obtain the solution at any other points at the same time.

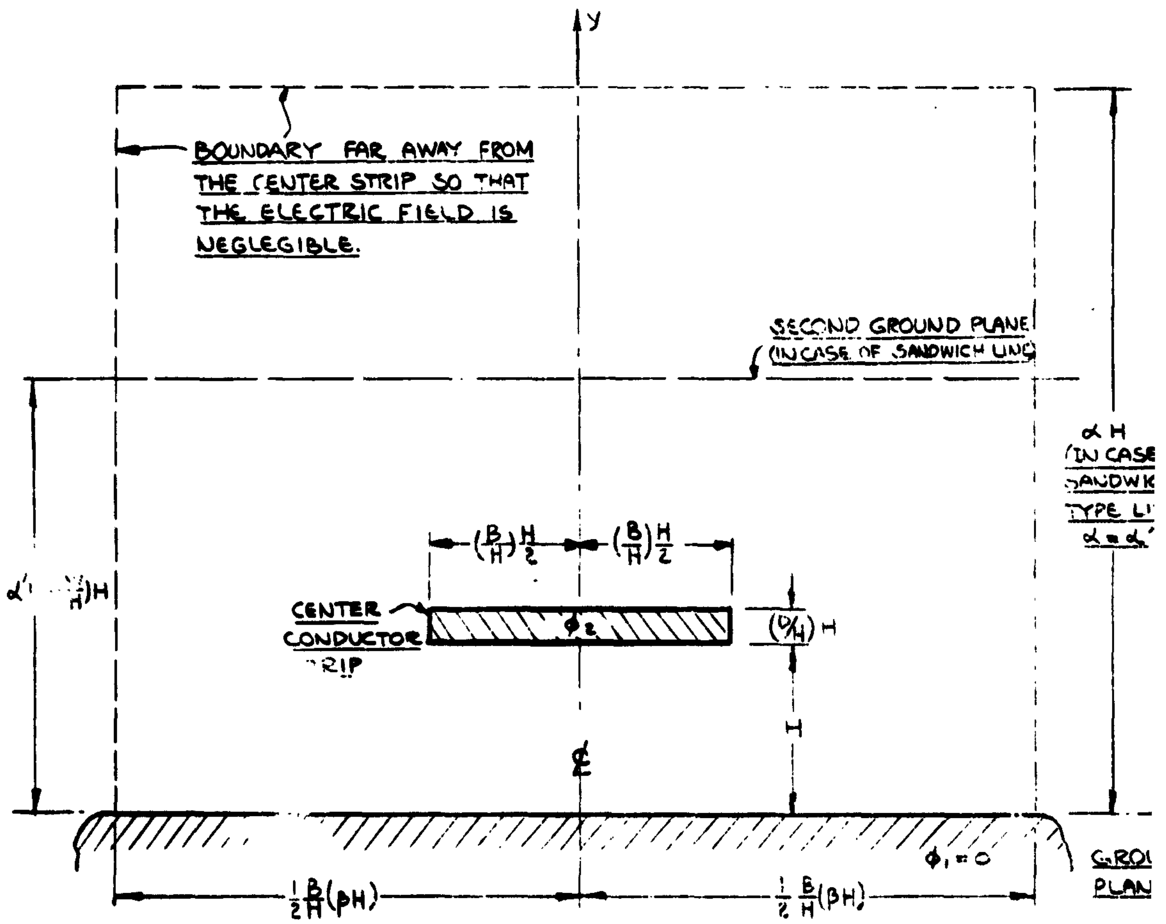


Fig. 1

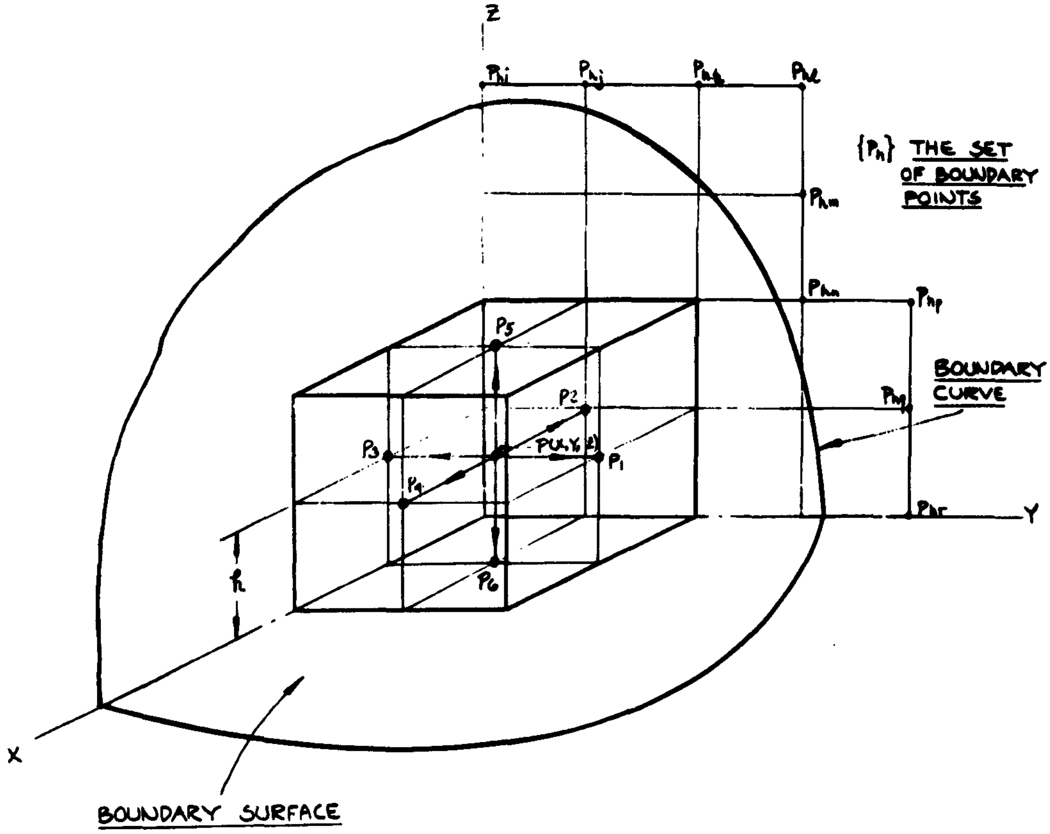


Fig. 2

DESCRIPTION OF MONTE CARLO METHOD FOR SOLVING THE LAPLACE'S EQUATION

It may be helpful to describe the well known Monte Carlo Method briefly. Given a Poisson's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = F\ (x, y, z)$$

where F (x, y, z) is a function specified within a closed boundary surface S, V(x, y, z) is the potential at the point whose coordinates

are x, y, z. On the boundary surface S, the value of the potential function, V, is given.

Consider that the region inside the surface, S, is divided up into small cubical lattices of sides h. And the boundary surface is replaced by a set of boundary points $\{P_h\}$ as shown in Fig. 2. This set of points, $\{P_h\}$ includes all nearest exterior neighbors to all lattice points within S and the nearest exterior points diagonally across the lattice points in S. The values of the potential function V at all boundary points $\{P_h\}$ are given as $\{\phi(x_h, y_h, z_h)\}$. Then the poisson's equation may be approximated as the difference equation

$$\Delta_x V + \Delta_y V + \Delta_z V = F(x,y,z) \quad \ldots\ldots (1)$$

where

$$\Delta_x V = \frac{1}{h^2}\left\{V(x+h,y,z) + V(x-h,y,z) - 2V(x,y,z)\right\}$$

$$\Delta_y V = \frac{1}{h^2}\left\{V(x,y+h,z) + V(x,y-h,z) - 2V(x,y,z)\right\}$$

$$\Delta_z V = \frac{1}{h^2}\left\{V(x,y,z+h) + V(x,y,z-h) - 2V(x,y,z)\right\}$$

Denote the point whose coordinates are x, y, z as P and its closest neighboring points as $P_1, P_2 \ldots\ldots P_6$. Equation (1) becomes:

$$V(p) = \frac{1}{6}\left[\sum_{i=1}^{6} V(p_i) - h^2 F(p)\right] \quad \ldots\ldots (2)$$

Note that the difference equation (2) gives the relation between the potential at any point P and those at its adjacent points. In the Monte Carlo Method, we shall consider the coefficients of the difference equation as transition probabilities from one point P to its six

closest neighbors. Then for a set of fictitious particles taking random steps at the point P, with the directions of their steps governed by random sampling technique, the probability for these particles to step to any one of the neighboring points is given by the coefficient of the difference equation. In our case, the particles have equal probability to step to any one of the six nearest neighboring points.

Now consider a particle is allowed to take X random walks where all walks starts at a lattice point P_o and terminates whenever the particle reaches the boundary. As the particle is momentarily at a point P, the probability that it will step to each one of the six closest neighbors of P is 1/6. A tally of the value of the potential function made for each walk will depend on the transition probability between successive points and the value of F(P) at the points which the particle reached in the walk. It also depends on the value of the potential at the boundary point, $\phi(x_h, y_h, z_h)$, where the walk is terminated. Here for the j^{th} walk, the expected value of the tally Z_j is given as

$$Z_j = -\sum_{i} \frac{1}{6} h^2 \cdot F(P_i) + \phi(x_j, y_j, z_j) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where $\phi(x_j, y_j, z_j)$ is the boundary point at which the j^{th} walk is terminated and P_i's are points reached in the walk. The solution of equation (2) at the point P. can be estimated as:

$$\bar{V}(p_o) = \frac{1}{X} \sum_{j=1}^{X} Z_j$$

And the error of the solution is estimated by the variance

$$C^2\ (\bar{V}) = \frac{1}{X} \sum_{j=1}^{X} (Z_j - \bar{V}^2)$$

It is obvious here that the larger X is, the higher the accuracy will be.

As mentioned in the introduction, the analysis of the microstrip line is simplified as a two-dimensional problem. Also, with the dielectric being homogeneous and lossless and the conductors being perfect, the function F(p) in the poisson's equation is zero at all points in the boundary. Hence equations (2) and (3) become:

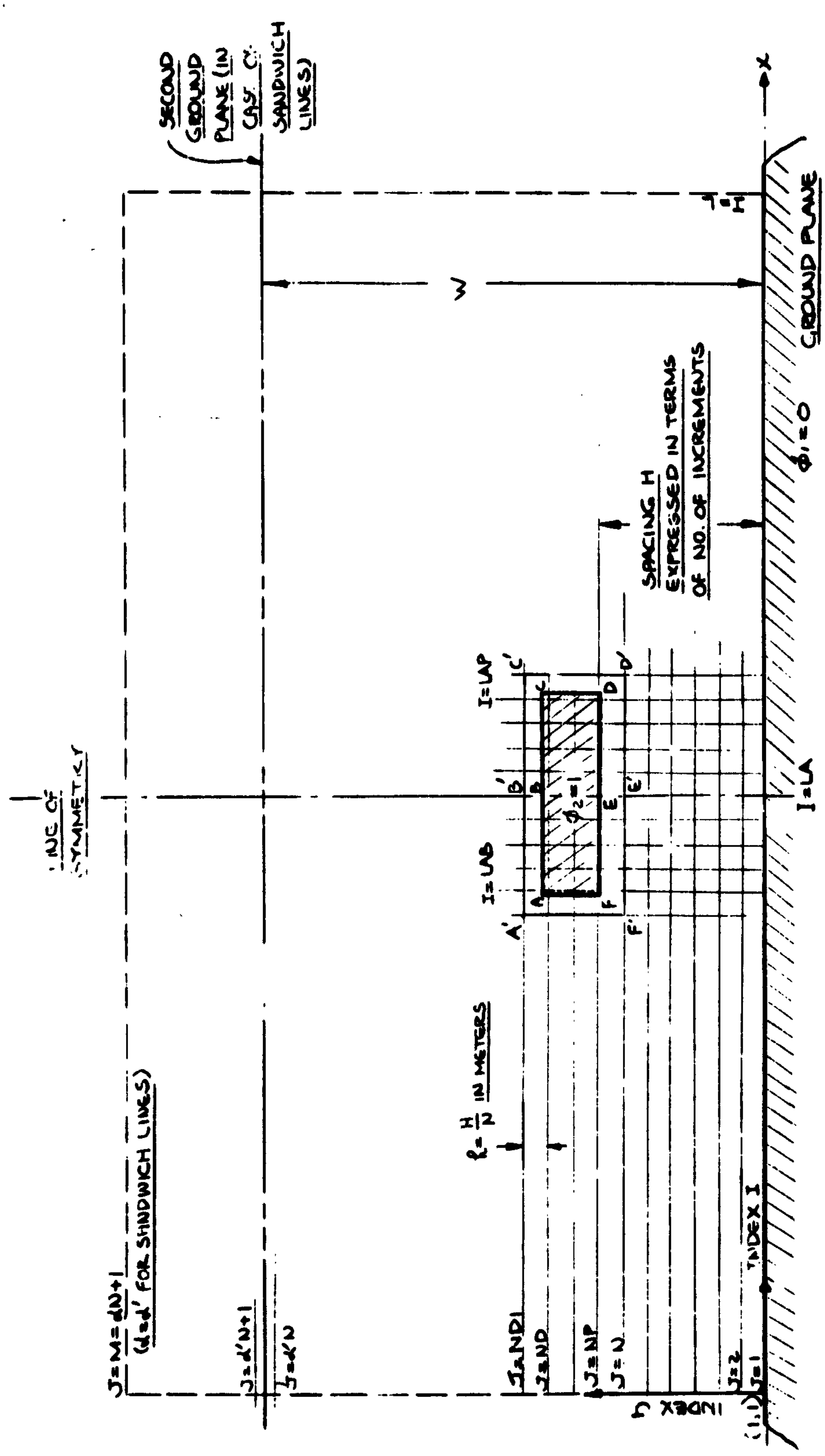
$$V\ (P) = \frac{1}{4} \sum_{i=1}^{4} V\ (P_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

And

$$Z_j = \phi\,(x_j,\ y_j) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

DESCRIPTION OF THE COMPUTATION PROCEDURE AND FLOW CHART

First let us redraw the cross-section of a microstrip line in Fig. 3, where all dimensions all indicated by the number of meshes of sides h. It is clear that h is equal to $\frac{H}{N}$. The coordinates of each of the boundary points are indicated in this figure in terms of the indices I and J which denote the lattice point representing the boundary according to the rule described. Obviously, from the rule, ($\alpha^1 N$) is the largest integer $\leq \frac{W}{H} N$. $(\frac{BN}{2H})_I$ and $(\frac{DN}{H})_I$ are the largest integers equal to or smaller than $\frac{BN}{2H}$ and $\frac{DN}{H}$ respectively. With β being large and arbitrary, we can always make $\beta \frac{BN}{2H}$ an integer.



NOTE:

$M = \alpha N + 1$ OR $\alpha' N + 1$

$LA = \beta(\frac{B}{2H}N) + 1, \quad L = \beta \frac{B}{H} N + 1$

$NP = N + 1$

$ND = N + (\frac{P}{H}N)_I + 1$

$NDI = ND + 1$

$LAP = LB + (\frac{B}{2H}N)_I$

$LAB = LB - (\frac{B}{2H}N)_I$

FIGURE 3

It may also be helpful to list here a few well known relations which give us the parameters of interest in terms of the scaler potential, V.

The electric field intensity, $E = -\,\mathrm{Grad}\ V$ .... (a)

The charge density .... $\vec{D} = \epsilon\,\vec{E}$ .... (b)

The total surface charge on the strip per unit length, q,

$$= \int_{ACDFA} (\vec{D}\cdot\vec{n})\,ds \quad \text{........ (c)}$$

(where $\vec{n}$ is a unit vector pointing outward of and being perpendicular to the surface of center conductor.)

Therefore, $q \propto \phi_2$

The characteristic impedance, $Z_o = \dfrac{(\mu\epsilon)^{\frac{1}{2}}}{q}$ .... (d)

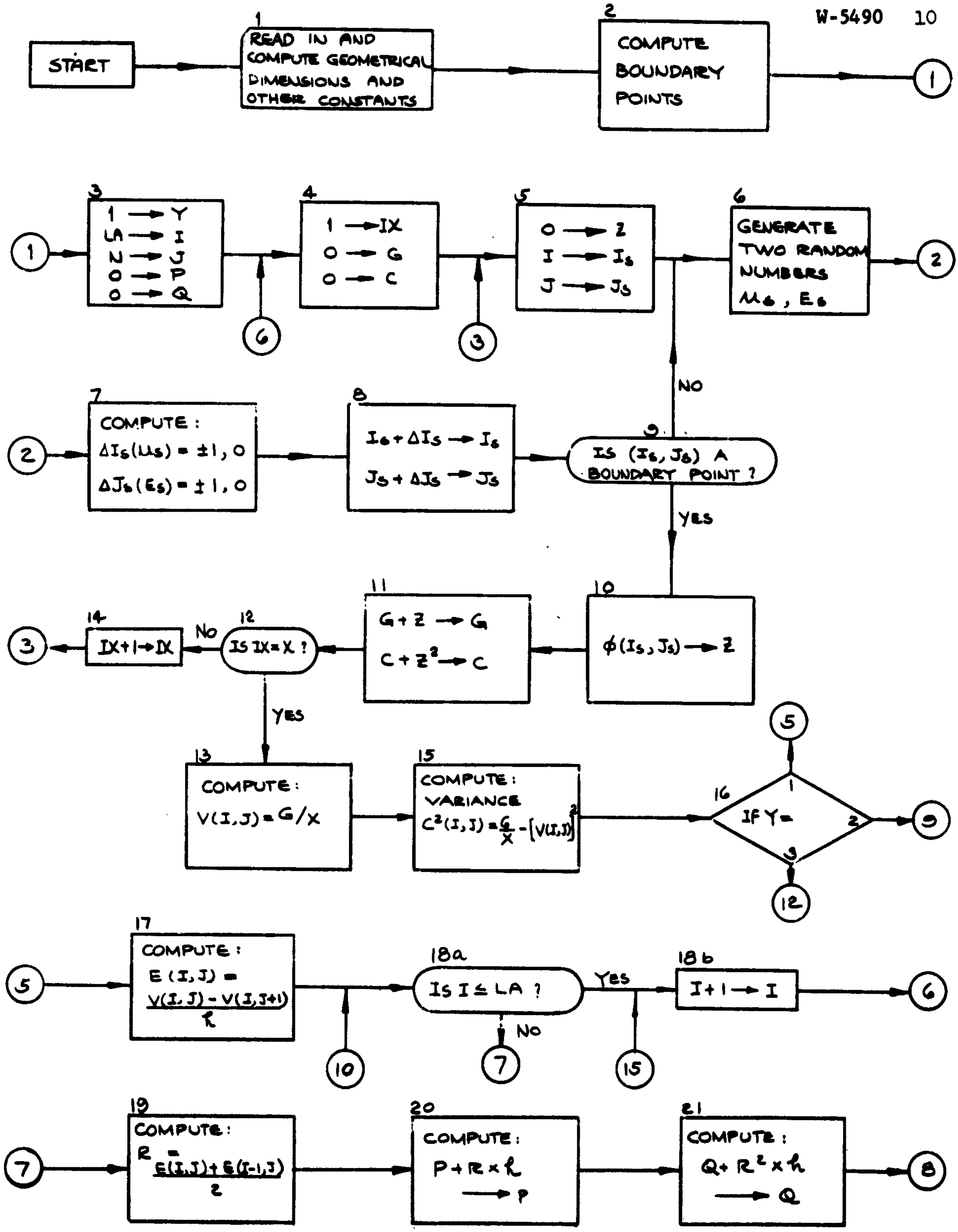
Alpha = max. possible power loss/power transmitted

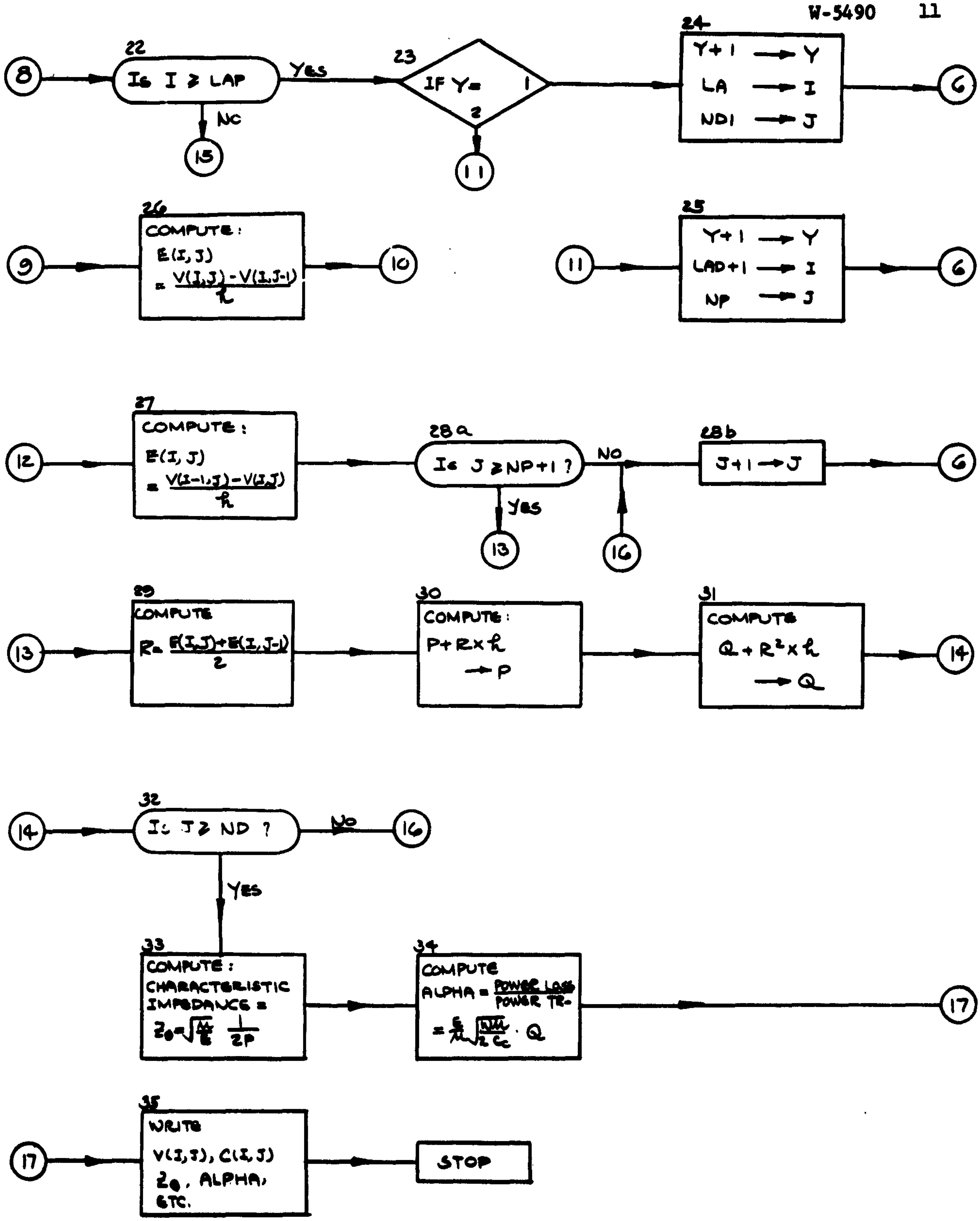
$$= \frac{1}{2Z_o}\sqrt{\frac{\omega\mu}{2\sigma}}\left(\frac{\epsilon}{\mu}\right)\int_{ACDFA} |E|^2 ds$$

$$= \frac{\phi_2^2}{2Z_o}\sqrt{\frac{\omega\mu}{2\sigma}}\left(\frac{\epsilon}{\mu}\right)\int_{ACDFA} |E'|^2 ds \;,\quad |E'| = \frac{|E|}{\phi_2} \quad \text{.... (e)}$$

From these relations we can derive all parameters of interest once the electric potential at points just outside of the center strip is known. Because of the symmetry of the geometry of the line, it is sufficient to compute the potential at points whose coordinates are: 1) from I = LA to I = LAP and J = N, 2) from I = LA to I = LAP and J = NDI and 3) I = LAP+1 and J = N to J = ND1.

We shall describe the flow chart in detail.


START
1 READ IN AND COMPUTE GEOMETRICAL DIMENSIONS AND OTHER CONSTANTS
2 COMPUTE BOUNDARY POINTS
3 1 → Y
LA → I
N → J
0 → P
0 → Q
4 1 → IX
0 → G
0 → C
5 0 → Z
I → Is
J → Js
6 GENERATE TWO RANDOM NUMBERS μs, Es
7 COMPUTE:
ΔIs(μs) = ±1, 0
ΔJs(Es) = ±1, 0
8 Is + ΔIs → Is
Js + ΔJs → Js
9 IS (Is, Js) A BOUNDARY POINT ?
NO
YES
10 φ(Is, Js) → Z
11 G + Z → G
C + Z² → C
12 IS IX = X ?
NO
YES
14 IX+1 → IX
13 COMPUTE:
V(I,J) = G/X
15 COMPUTE:
VARIANCE
C²(I,J) = G/X − [V(I,J)]²
16 IF Y =
1
2
3
17 COMPUTE:
E(I,J) = (V(I,J) − V(I,J+1))/h
18a IS I ≤ LA ?
YES
NO
18b I+1 → I
19 COMPUTE:
R = (E(I,J) + E(I-1,J))/2
20 COMPUTE:
P + R × h → P
21 COMPUTE:
Q + R² × h → Q


8
22
Is I ≥ LAP
Yes
No
15
23
IF Y=
1
2
11
24
Y+1 → Y
LA → I
ND1 → J
6
9
26
COMPUTE:
E(I,J)
= (V(I,J) - V(I,J-1)) / h
10
11
25
Y+1 → Y
LAD+1 → I
NP → J
6
12
27
COMPUTE:
E(I,J)
= (V(I-1,J) - V(I,J)) / h
28a
Is J ≥ NP+1 ?
NO
Yes
13
16
28b
J+1 → J
6
13
29
COMPUTE
R = (E(I,J)+E(I,J-1)) / 2
30
COMPUTE:
P+R×h
→ P
31
COMPUTE
Q + R²×h
→ Q
14
14
32
Is J ≥ ND ?
No
16
Yes
33
COMPUTE:
CHARACTERISTIC
IMPEDANCE =
Z0 = √(μ/ε) 1/2P
34
COMPUTE
ALPHA = POWER LOSS / POWER TR.
= ... Q
17
17
35
WRITE
V(I,J), C(I,J)
Z0, ALPHA,
ETC.
STOP

Box 1. Input data:

a) Dimensional constants: H (in meters), $\frac{B}{H}$, $\frac{D}{H}$, N, and $\frac{W}{H}$ (for sandwich lines only)

b) Number of walks to be made, X

c) Arbitrarily chosen constants α (for open line) and β

d) Dielectric constants, ϵ , permeability μ, conductivity of the conductors σ_c, and frequency at which the power loss is of interest, .

Box 2. a) Compute constants and boundary points LA, NP, L, etc. and express them in fixed point numbers conforming with the rule mentioned above.

Box 3. Register Y is set to one, and solution point P (I,J) is assigned to be P (LA,N). Register, P, for integral of electric field strength E and Register,Q, for integral of the square of the electric field strength, E^2, are set to be zero.

Box 4. Sum register, G, for tally Z_i and sum of square register C for tally Z_i^2 are set to zero. The counter IX indexing the walks is set to one.

Box 5. Current point counters I_s and J_s are set to the indices of the solution point P(I,J). Register Z for tally during a walk is set to zero.

Box 6. Two random number U_s and E_s are generated. Here the sub-routine RANF already in Fortran Library which generates floating random numbers of magnitude between 0 and 1 with even distribution are used.

Box 7. Compute ΔI_s and ΔJ_s according to the table below:

μ_s	E_s	I_s	J_s
< 0.5	≥ 0.5	1	0
< 0.5	< 0.5	0	1
≥ 0.5	≥ 0.5	0	-1
≥ 0.5	< 0.5	-1	0

Box 8. The current point are advanced to where $I = I_s + \Delta I_s$, $J_s = J_s + \Delta J_s$

Box 9. Test if the new current point coincides with any of the boundary points. If it does not, step back to Box 6. If it does, proceed to Box 10.

Box 10. The boundary value at the boundary point reached is transferred to the tally register Z.

Box 11. The content of the tally register Z is added to that of the sum register G. And the square of the content of register Z is added to that of the sum of square register C.

Box 12. If N walks are completed, proceed to Box 13. Otherwise, step to Box 14 where the register IX is increased by one. Then step back to Box 5 to start a new walk.

Box 13. Estimate of the solution at the point (I, J) is computed and store in V(I, J).

Box 15. The variance of the estimated solution is computed.

Box 16. If the content of the register Y is one (solution points are on line E'D' in Fig. 3, proceed to Box 17. If it is two (solution points are on line B'C') go to Box 26. If it is 3 (solution in points are on line C'D') go to box 27.

Box 17. Compute the electric field strength using the approximated equation $E\ (I,J\ ) = \frac{V(I,J)\ -\ V(I,\ J+1)}{h}$. Here (on the line E'D') the x-directional electric field is zero. V(I, J+1) is given as boundary value.

Box 18. If I is equal to LA, increase the register I by one and step back to Box 4 to compute the solution for the next point. Otherwise, with solutions for more than one point are available, go to Box 19.

Box 19. Compute the average field strength between the two adjacent points (I, J) and (I-1, J).

Box 20. Compute the line integral $\int_{ACDFA} |E|\ ds$ by using the trapezoidal rule of integration.

Box 21. Compute the line integral $\int_{ACDFA} |E|^2\ ds$ by using the trapezoidal rule of integration.

Box 22. If the solutions for all points with coordinates I = LA to I = LAP and J = N are obtained, proceed to Box 23. If otherwise, stop back to Box 18b.

Box 23. If register Y contains the number one, go to Box 24. If it contains the number two, go to Box 25.

Box 24. Solution point registers I and J are assigned to be LA and ND1. And the register Y is increased by one. Then, step back to Box 4.

Box 26. Compute electric field strength at points on the line B'C' in Fig. 3. Here the point (I, J-1) is on the surface of the center strip.

Box 27. Compute electric field strength at point on line C'D' where the vertical electric field is zero (I-1, J) are boundary points.

Box 28. If J is smaller than NP+1 (the solution for only one point on the line C'D' is available), step back to Box 28b where the register J is increased by one. Then, step back to Box 4 to compute the solution of another point. If otherwise, proceed to Box 29.

Boxes 29, 30 & 31. Same as boxes 19, 20, and 21 respectively.

Box 32. If the solution of all points are obtained proceed to Box 33. If not, step back to 28b.

Box 33. Compute the characteristic impedance of the line.

Box 34. Compute the $\left(\frac{\text{Max. possible power loss}}{\text{power transmitted}} = \text{Alpha}\right)$

Box 35. Print out the estimated potential at all points on the lines B'C', C'D' and D'E with the variances of the estimated solutions. The characteristic impedance of the line and the ratio ALPHA.

LIMITATION OF THE PROGRAM

Due to the limitation of the storage space, using an IBM 7090 computer with 32K words storage, the maximum number of meshes

the dimensions H, B/H and D/H may be divided into are as follows:

$$\alpha N \leq 2,000$$

$$\beta \frac{BN}{H} \leq 6,000$$

Therefore $H = 2,000/\alpha$, $B/H = 6,000/(\beta N)$

This program was used to estimate the characteristic impedance of open strip lines of the dimensions: B/H = 1.0, 2.0, and 3.0 and D/H = 0.3. H was divided into ten increments. Both α and β were chosen to be ten. For one hundred random walks, the results obtained are within 10% of the experimentally measured values. Although dividing the region into fewer meshes and increasing the number of walks will certainly improve the accuracy, the computing time have been found to be extremely high (say, one hour on the IBM 7090).

More accurate results will be obtained in a reasonable amount of computing time in the cases where the lines are the sandwich type. The reason is that with the spacing between the two horizontal boundary lines at ground potential being not more than two to five times the spacing H, the mean distance of the random walks is shorter; therefore each walk will take less time.

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